

## MATH 121

### Functions—Function’s Definition

1. **(Section 1.2)** For each part give an example of a function, if one exists, with the set of inputs (domain) =  $A$  and the possible outputs coming from  $B$ , satisfying the stated condition. (Note: For each function, your definition must say what the function does to each input. i.e. For each input what is the output?) Note:  $\mathfrak{R}$  is the set of real numbers.
  - a)  $A = \{1, 2, 3\}$ ,  $B = \{\sqrt{7}, 1, 0, 4\}$  and  $f$  has exactly 2 outputs.
  - b)  $A = \{\text{red, blue}\}$ ,  $B = \mathfrak{R}$ , the real numbers, and (i)  $f$  has exactly 1 output. (ii)  $f$  has exactly two outputs. (iii)  $f$  has 3 outputs.
  - c)  $A = \mathfrak{R}$ ,  $B = \{0, 1\}$  and  $f$  has exactly 2 outputs. (Give 2 examples.)
  - d)  $A = \{1, 2, 3\}$ ,  $B = \{a, b, c, d\}$  and no two inputs have the same output. (Give two examples) (Remark: Such a function is called “one – to – one.”)
  - e)  $A = \mathfrak{R}$ ,  $B = \{x \mid x \text{ is a real number and } x \geq 0\}$  and every number in  $B$ , “gets hit” by some number in  $A$ . (Remark: Such a function is called “onto”.)
  
2. **(Section 1.2)** Consider a function  $f$  with the following properties
  - i) Domain of  $f$  is all real numbers,  $\mathfrak{R}$ .
  - ii) Range of  $f$  is the interval  $[-1, 1]$ .
  - iii)  $f(-x) = f(x)$  for all inputs  $x$  ( $f$  is called an “even” function)
  - a) Describe  $f$  with a formula (or formulas)
  - b) Describe  $f$  verbally. (The function should be different than in part (a))
  - c) Describe  $f$  with a table (different function again.)
  
3. **(Section 1.2)** Being able to recognize patterns is an invaluable problem-solving technique, and it helps immensely in the study of the concept of a function. In table below find a formula for  $f(x)$ .

$x$	1	2	3	4	-1	-2	-3
$f(x)$	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{7}{4}$	2	undefined	8	$\frac{11}{2}$

4. **(Section 1.2)** Sketch the graph of a function  $f$  having the given domain,  $D$ , and range  $R$ .
  - a)  $D = \mathfrak{R}$ ;  $R = (-3, 3)$
  - b)  $D = (-4, 4)$ ;  $R = [-3, 5]$
  - c)  $D = [-2, 3)$ ;  $R = \mathfrak{R}$
  - d)  $D = \mathfrak{R}$ ;  $R = \{-1, 1\}$
  
5. **(Section 1.2)** Find the function  $f$  such that  $f(x+3) = f(x)$  for all numbers  $x$ .
  
6. **(Section 1.5)** A piecewise-defined function  $f$  has a domain of  $[0, 6]$  and a range of  $(2, 5)$ . Give a possible formula for this function  $f$ . (Hint: Draw a picture...be creative.)

7. (Section 1.6) Give a sketch of the graph of a single function  $f$  which meets ALL of the following conditions:
- The domain of  $f$  is  $[-3, 8]$
  - The range of  $f$  is  $(-1, 6)$
  - $f(-2) = f(1)$
  - $f(1) = \frac{1}{f(-3)}$
  - $f(5) = 2f(-1)$
8. (Section 1.6) Give a sketch of a graph of ONE function meeting ALL of the following conditions:
- The domain of  $f$  is  $(-\infty, -1] \cup (4, +\infty)$
  - The range of  $f$  is  $[0, +\infty)$
  - $f(-3) \cdot f(6) = 10$
  - $f(-3) + f(-1) = 1$
  - $f(-4) = 2 + f(7)$
  - $f(-2) = \sqrt{f(-4)}$
9. (Section 3.4) For each of the following, give an example of a polynomial function that satisfies the given condition(s):
- $f$  has degree 2 and  $f(0) = 2$
  - $f$  has degree 3,  $f(0) = 2$  and  $f(1) = 4$
  - $f$  has degree 1,  $f(0) = 2$  and  $f(1) = 5$
10. (Section 3.4) Give an example of a cubic polynomial function  $f$ , if one exists, whose graph has:
- 3  $x$  – intercepts
  - 2  $x$  – intercepts
  - 1  $x$  – intercepts
  - 0  $x$  – intercepts
11. (Section 3.4) Give an example of a 4<sup>th</sup> – degree polynomial function  $P$  (with real coefficients), if one exists, which has:
- Exactly 4 distinct real roots, and each root of multiplicity 1.
  - Exactly 3 distinct real roots, and each root multiplicity 1.
  - Exactly 2 distinct real roots, and each root of multiplicity 1.
  - Exactly 1 distinct real root, and this root is of multiplicity 1.
  - Exactly 2 complex roots.

12. (Section 3.4) Give an example of a polynomial function  $f$  such that
- $f(x) > 0$  if  $|x| > 4$  or  $|x| < 1$ , or if  $-4 < x < -1$  or  $1 < x < 4$ .
  - $f(x) > 0$  if  $x < -4$  or  $-1 < x < 1$  or  $x > 1$ ,  $f(x) < 0$  if  $-4 < x < -1$  and  $F(x) = 0$  otherwise.
13. (Section 3.5) Define a rational function  $f$  such that
- $x = 1$  is a vertical asymptote
  - $y = 5$  is a horizontal asymptote
  - $y$  - intercept is 5
  - There are no  $x$  - intercepts
14. (Section 3.5) Give two examples of a rational function with  $y = 1$  as an asymptote and passes through the point  $(2, 2)$ .

**Answer Keys:**

1. Answers vary.

a)  $\{(1,1), (2,0), (3,0)\}$

b) (i)  $\{(red, 2), (blue, 2)\}$  (ii)  $\{(red, 1), (blue, 0)\}$  (iii) not possible

c)  $f(x) = \begin{cases} 0 & x \geq 0 \\ 1 & x < 0 \end{cases}$

d)  $\{(1, a), (2, b), (3, d)\}$

e)  $f(x) = x^2$

2. Answers vary.

a)  $f(x) = \begin{cases} 1 & x \geq \sqrt{2} \\ x^2 - 1, & -\sqrt{2} < x < \sqrt{2} \\ 1 & x \leq -\sqrt{2} \end{cases}$

3.  $f(x) = \frac{3x-2}{x+1}$

4. Answers vary.

5. Answers vary,  $f(x) = 4$ . ☺

6. Answers vary. Have fun with this one.

7 and 8. Answers vary, but make sure to give one function that satisfies all conditions.

9. (a)  $f(x) = x^2 + 3x + 2$

(b)  $f(x) = x^3 + 2x^2 - x + 2$

(c)  $f(x) = 3x + 2$

10. (a)  $f(x) = (x-3)(x+9)(x+2)$   
(b) not possible  
(c)  $f(x) = (x-3)(x^2+1)$   
(d) not possible

11. Answers vary. (a)  $f(x) = (x-3)(x-5)(x+8)(x+5)$   
(b) not possible  
(c)  $f(x) = (x-4)(x+3)(x-3i)(x+3i)$   
(d) not possible  
(e)  $f(x) = (x-2i)^2(x+2i)^2$

12. Answers vary.

13. Answers vary. One possible answer is  $f(x) = \frac{5x^2+5}{x^2-2x+1}$ .

14. Answers vary. One possible answer is  $f(x) = \frac{3x+4}{3x-1}$ .