

MATH 121
Functions—Functional Notation

1. (Section 1.2) If $g(x) = -2x + 7$, find and simplify completely:

- a) $g(0)$ b) $g(1)$ c) $g(-2)$ d) $g(4)$
- e) $g(a)$ f) $g(\odot)$ g) $g(x+h)$

2. (Section 1.2) If $f(x) = \frac{x^2 - 2}{x + 2}$, find and simplify completely:

- a) $f(1)$ b) $f(-1)$ c) $f(0)$ d) $f(-3)$
- e) $f(a)$ f) $f(\odot)$ g) $f(x+h)$

3. (Section 1.2) Suppose f is the function given by $f(x) = -x^2 + 2x - 5$. Simplify each of the following completely.

- a) $\frac{f(x+h) - f(x)}{ax}$ b) $\frac{f(w) - f(1)}{1-w}$ c) $\frac{f(-2)}{3} + \frac{f(6)}{2}$
- d) $\frac{f(x^2) - f(2x)}{xf(x)}$ e) $\frac{f(x+1) - f(x-1)}{2x}$

4. (Section 1.2) Let g be the function given by $g(x) = \frac{2}{x}$. Find and simplify:

$$\frac{g(2+h) - g(2)}{h}.$$

5. (Section 1.2) For a linear function f , $f(2) = 4$ and $f(-1) = 3$.

- a) Find the function.
b) Find $f(3)$.
c) For what x is $f(x) = -100$?

6. (Section 1.2) Suppose f is a **linear** function, and it is known that $f(-1) = 2$ and $f(2) = -5$.

- (a) Find $f(9)$.

(b) For what input x is $f(x) = \frac{35}{4}$?

7. (Section 1.3) For a linear function f , $f(-1) = 3$ and $f(2) = 5$.

- a) Find a formula for $f(x)$.
b) Write two other points for this function.
c) Determine the intercepts of the graph of f .
d) Determine the exact value of $f(7.5)$.
e) For what x is $f(x) = -10\frac{3}{5}$?

8. (Section 1.6) Consider the functions $p(x) = -x^2$ and $q(x) = \sqrt{19 - 3x}$

- What is the domain of q ?
- Simplify completely: $-4 \cdot p(-4)$
- Simplify completely: $\sqrt{\frac{q(5)}{5 - p(-2)}}$
- Simplify completely: $[q(-2) - q(-10)]^2$
- Simplify completely: $\frac{p(6) + 6}{p(2) + 2}$
- Simplify completely: $\frac{p(x+h) - p(x)}{2h}$

9. (Section 2.3) Suppose p is the function $p(x) = 2x + \frac{20}{x} - 5$.

- For what input(s) x is $p(x) = 8$?
- For what input(s) x is $p(x) = 36$?
- Solve the following equation: $p(2x) = p(x)$

10. (Section 2.3) Suppose f and g are the functions given by $f(x) = x - 5$ and $g(x) = \frac{x-8}{x}$. Solve each of the following equations.

- $f(x) + g(x) = f(8)$
- $f(2x+1) - f(3-x) = g(-6)$
- $g(-4) \cdot g(x+8) = g(-8) \cdot g(-x+8)$

11. (Section 2.5) Suppose f is the function $f(x) = 3 - \frac{1}{3}x$ and g is the function $g(x) = -x + 4$. Solve for x in each of the following equations.

(a) $|f(x) + 2| = \frac{g(-2)}{2}$

(b) $5 - \frac{6|g(x)|}{5} = f(0)$

(c) $|f(2x)| = |g(-x)|$

12. (Section 4.1) Let $f(x) = 3 + x + x^3$,

- Find $f^{-1}(5)$
- Graph f by plotting points.
- Graph f^{-1} ; comment on the relationship between the graph of f , f^{-1} , and $y = x$.

13. (Section 3.6) Let $f(x) = \frac{x-4}{2x^3 - x^2 - 8x + 4}$

- Find all x such that $f(x) > 0$.
- Find all x such that $f(x) < -1$.

14. (Section 3.5) Let $f(x) = \frac{x^2}{x-1} + 1$ and $g(x) = \frac{4x-2}{x-2} + \frac{x+4}{2}$

- Find the x -intercepts of the graph of f .
- Find the x -intercepts of the graph of g .
- Find the value(s) for x , if any exist, for which $f(x) = g(x)$.

15. (Section 3.6) Suppose f and g are functions such that $f(x) = \frac{x-5}{2} - \frac{3x-1}{4}$ and

$g(x) = \frac{1}{3} - \frac{3}{5}x$, solve each of the following:

- $f(0) > g(x)$
- $f(x) \leq g\left(\frac{-5}{3}\right)$

16. (Section 3.5) Let $f(x) = \frac{x}{x-2}$ and $g(x) = x^2 + 3$. Find and simplify completely:

a) $\frac{(g \circ f)(3)}{(f \circ g)(3)}$ b) $\frac{g(x+2) - g(x)}{x \cdot g(1)}$

17. (Section 4.1)

a) Let f be a function such that $f(2) = 4$ and $f(8) = 0$. If g is the inverse function of

f , find and simplify: $\frac{(f \circ g)(4)}{g(0)} + (g \circ f)(1)$

b) Find the inverse of the function: $f(x) = \sqrt[3]{\frac{4+3x}{x}}$

18. (Section 4.1) Suppose f and g are one-to-one functions such that $f(2) = 7$, $f(4) = 2$, and $g(2) = 5$. Find the value, if possible, of

- $(g \circ f^{-1})(7)$
- $(f \circ g^{-1})(5)$
- $(f^{-1} \circ g^{-1})(5)$
- $(g^{-1} \circ f^{-1})(2)$
- $(f \circ f^{-1})(7)$

$$f) (f^{-1} \circ f)(8)$$

$$g) (g \circ g)(2)$$

Answer Keys:

1. (a) 7 (b) 5 (c) 11 (d) -1 (e) $-2a + 7$ (f) $-2\odot + 7$ (g) $-2(x+h) + 7$

2. (a) $\frac{-1}{3}$ (b) -1 (c) -1 (d) -7 (e) $\frac{a^2 - 2}{a + 2}$ (f) $(\odot^2 - 2)/(\odot + 2)$

(g) $\frac{(x+h)^2 - 2}{(x+h) + 2}$

3. (a) $\frac{-2xh - h^2 + 2h}{ax}$ (b) $w - 1$ (c) 11 (d) $\frac{x^3 - 6x + 4}{x^2 - 2x + 5}$

(e) $\frac{-2x + 2}{x}$

4. $\frac{-1}{2+h}$

5. (a) $y = \frac{x}{3} + \frac{10}{3}$ (b) $\frac{13}{3}$ (c) -310

6. (a) $f(9) = 12$ (b) $x = \frac{47}{4}$

7. (a) $f(x) = \frac{2}{3}x + \frac{11}{3}$ (b) $\left(0, \frac{11}{3}\right), \left(1, \frac{13}{3}\right)$ (c) $\left(0, \frac{11}{3}\right), \left(\frac{-11}{2}, 0\right)$

(d) $\frac{-196}{10}$

8. (a) $\left[-\infty, \frac{19}{3}\right]$ (b) 64 (c) $\frac{2}{3}$ (d) 4

(e) 15 (f) $\frac{-2x - h}{2}$

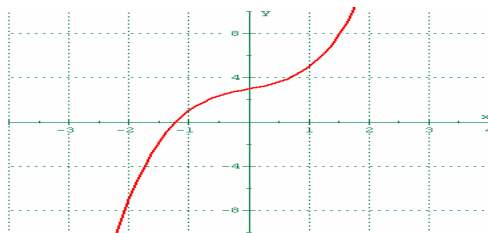
9. (a) $x = \frac{5}{2}, 4$ (b) $x = \frac{1}{2}, 20$ (c) $x = \pm\sqrt{5}$

10. (a) $x = 8, -1$ (b) $x = \frac{43}{9}$ (c) $x = 0, 40$

11. (a) $x = 6, 24$ (b) $x = \frac{7}{3}$ (c) $x = -21$

12. (a) $x = 1$

(b)



13. (a) $(-\infty, -2) \cup \left(\frac{1}{2}, 2\right) \cup (4, \infty)$ (b) $\left(2, \frac{1+\sqrt{56}}{4}\right)$

14. (a) $\frac{-1 \pm \sqrt{5}}{2}$ (b) $-5 \pm \sqrt{13}$

15. (a) $\frac{155}{36} < x$ (b) $x \geq \frac{-43}{3}$

16. (a) 10 (b) $\frac{x+1}{x}$

17. (a) $\frac{3}{2}$ (b) $y = \frac{4}{x^3 - 3}$

18. (a) 5
(b) 7
(c) 4
(d) not possible
(e) 7
(f) 8
(g) not possible.